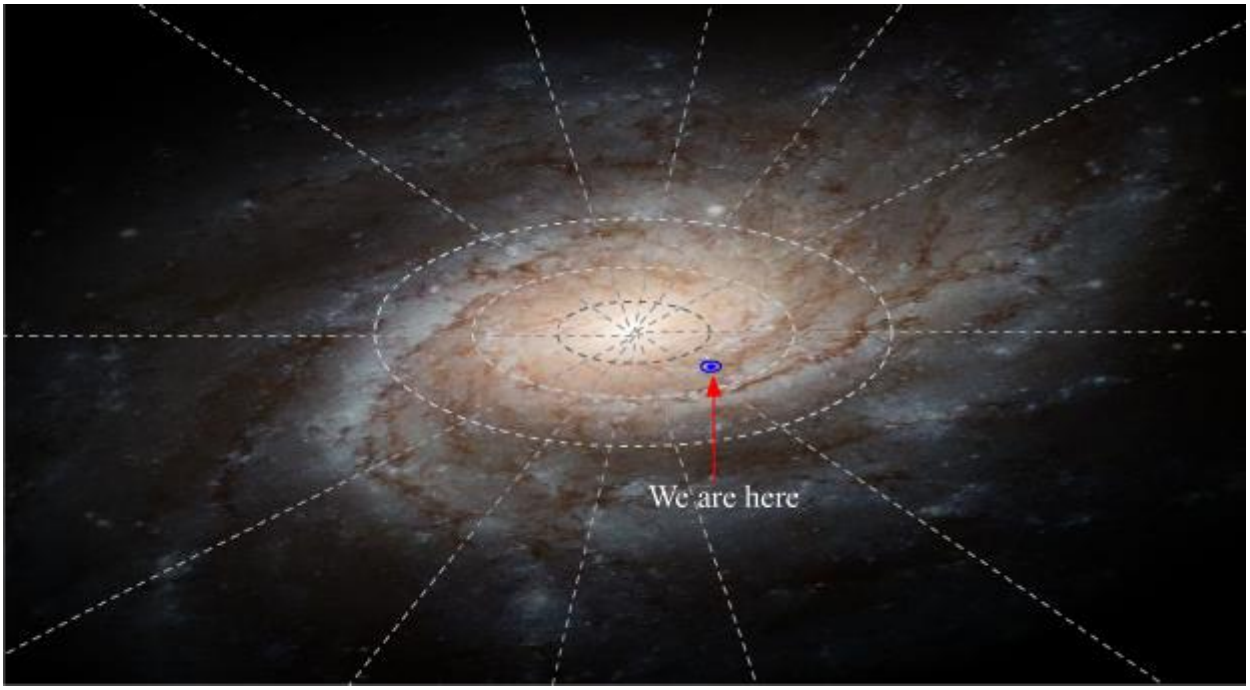
Gravitation

The Universal Law of Gravitation

Gravitation - An Overview



The picture that opens this lesson shows our location in the Milky Way galaxy. We are about 26,000 light years away from the centre of the galaxy. We are residing in this location from the very beginning of the Universe. This shows that there exists a powerful force that holds us in our place since the inception of the Universe. This force binds everything from stars to **galaxies** to **superclusters**

and is known as the **Gravitational force**.

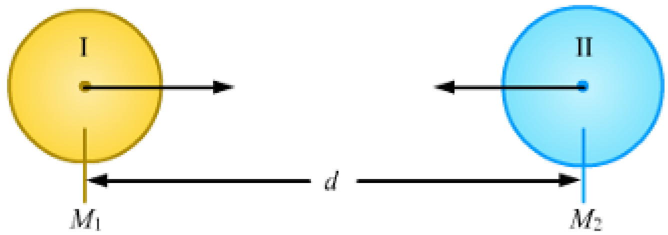
Universal Law of Gravitation

It is common to see things falling to the ground. The falling of a body to the ground is attributed to Earth's attraction for it. In fact, the weight of a body is expressed in terms of this force of attraction.

Newton's experiments showed that Earth's attraction, when at a constant distance from another body, varied directly with the mass of the other body. However, this was only a partial expression of the general law of gravitation. This law states that *every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them*.

All planets in the solar system are bound to revolve in their fixed orbits by the gravitational attraction of the sun. The same force of gravity acts between Earth and the moon, making the moon revolve around Earth in its fixed orbit.

Gravitational Force - Mathematical Form



Let two objects **I** and **II,** of masses M_1 and M_2 respectively, be placed at a distance d from each other. As per the law of gravitation, the following two assertions can be made about the force of gravity (F) between the two objects.

(a) The force of gravity between the two objects is directly proportional to the product of their masses. This is expressed as:

$F \propto M_1 \times M_2$

(b) The force of gravity between the two objects is inversely proportional to the square of the distance between them. This is expressed as:

$F \propto (1/ d^2)$ **(By Inverse-Square rule)**

On combining both the equations, we obtain:

$$F \propto \frac{M_1 M_2}{d^2} \text{ or } F = G\frac{M_1 M_2}{d^2}$$

Where, G is a constant called **Universal Gravitational Constant** or **Newton's constant**.

Did You Know?

Now, we know that the motion of celestial bodies present in the universe is primarily governed by the gravitational force. There are many facts like this.

These are:

- The gravitational force does not depend on the medium between two bodies.

- We, too, pull or attract Earth toward ourselves by the same gravitational force. However, owing to Earth's large size, the effect of this force is negligible.
- The moon has no atmosphere as the gravitational force exerted by it is very small.
- It is a common misconception that Earth's gravity ceases to exist beyond its atmosphere. Earth does exert gravitational force beyond its atmosphere, but the effect of this force is very less. It is the gravitational force present between the moon and Earth which keeps the moon in an orbit around Earth.

Universal Gravitational Constant

Universal Gravitational Constant (G) is a constant of proportionality. Its value is constant at all places in the universe. Its value does not depend on the medium between two bodies.

SI unit of G

The force of gravity (*F)* between two objects of masses M_1 and M_2, which are at a distance *r* from each other, is given as:

$$F = G\frac{M_1 M_2}{r^2} \text{ or } G = \frac{Fr^2}{M_1 M_2}$$

On substituting the SI units of the various quantities in this equation, we obtain: G = Nm^2/kg^2

Therefore, the SI unit of G is Nm^2/kg^2.

Value of G

Henry Cavendish found the value of **Universal Gravitational Constant,** G with the help of a very sensitive balance. Its value is **6.673×10^{-11} Nm^2/kg^2**

Consider two bodies, each having the mass 1 kg. They are placed at a distance 1 m from each other. Using the value of G, the force of attraction is given as:

$$F = G\frac{m_1 \times m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(1)^2} \mathrm{N}$$

$$\Rightarrow F = 6.67 \times 10^{-11} \mathrm{~N}$$

Solved Examples

Easy

Example 1:

How will the gravitational force between two objects change when the distance between them is doubled?

Solution:

Suppose two objects having masses m_1 and m_2 are placed at a distance r from each other.

The force of gravitation between them is given as:

$$F = G \times \frac{m_1 m_2}{r^2}$$

Where G is the gravitational constant

Let the distance between them be doubled.

So, new distance, $r' = 2r$

The new force of gravitation is computed as:

$$F' = G \times \frac{m_1 m_2}{r'^2} = G \times \frac{m_1 m_2}{(2r)^2} = \frac{1}{4} \times (G \times \frac{m_1 m_2}{r^2})$$

$$\therefore F' = \frac{1}{4} \times F$$

Thus, when the distance between two objects is doubled, the gravitational force between them becomes one-fourth its original value.

Medium

Example 2:

Calculate the force of gravitation exerted by Earth on a body of mass 2 kg lying near the ground.

(Given: Earth's mass = 6 × 10^{24} kg; Earth's radius = 6.4 × 10^3 km;
G = 6.7 × 10^{-11} Nm^2/kg^2)

Solution:

The force of gravitation is given as:

$$F = G \times \frac{m_1 m_2}{r^2}$$

We know that:

$G = 6.7 \times 10^{-11}\ Nm^2/kg^2$

Earth's mass, $m_1 = 6 \times 10^{24}$ kg

Mass of the body, $m_2 = 2$ kg

Distance (r) between the body and Earth $= 6.4 \times 10^3$ km

$$= 6.4 \times 10^3 \times 1000 \text{ m}$$

$$= 6.4 \times 10^6 \text{ m}$$

$$\therefore F = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24} \times 2}{(6.4 \times 10^6)^2} = 19.6 \text{ N}$$

Hard

Example 3:

If the masses of Earth and the sun were concentrated at their respective centres, then what would be the gravitational force of attraction between the two?

(Given: Mass of the sun = 2 × 10^{30} kg; Earth's mass = 6 × 10^{24} kg; Average distance between Earth and the sun = 1.5 × 10^8 km)

Solution:

The force of gravitation between Earth and the sun is given by the formula:

$$F = \frac{GM_S M_E}{R^2}$$

Here, gravitational constant, G = 6.7 × 10^{-11} Nm^2/kg^2

Mass of the sun, M_S = 2 × 10^{30} kg

Earth's mass, M_E = 6 × 10^{24} kg

Distance (R) between Earth and the sun = 1.5 × 10^8 km = 1.5 × 10^{11} m

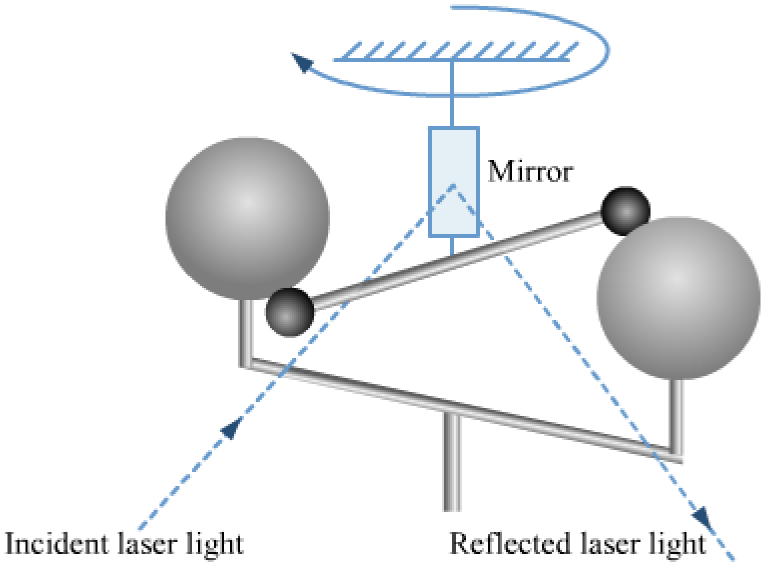
On substituting these values in the above formula, we get:

$$F = \frac{6.7 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{\left(1.5 \times 10^{11}\right)^2}$$

= 3.57 × 10^{22} N

Determination of 'G' by Henry Cavendish

The value of universal gravitational constant (G) was first determined by Henry Cavendish through the torsion bar experiment. The apparatus of this experiment comprises two pairs of spheres. Each pair of spheres forms a dumbbell having a common axis, as shown in the figure. One of the dumbbells is suspended from a quartz fibre. It rotates freely when the fibre is twisted. The position of a reflected light spot from a mirror attached to the fibre gives the measure of the amount of twists. The second dumbbell can be swivelled in such a way that each of its spheres is close to one of the spheres of the other dumbbell. The gravitational attraction between the two pairs of spheres twists the fibre and the magnitude of the force of gravity is calculated by measuring the amount of twists in the fibre.

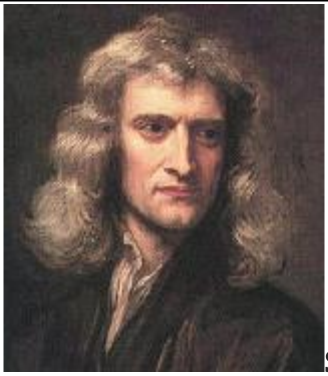


The value of G, as determined by Cavendish, came out to be **6.67×10^{-11} Nm^2/kg^2**.

Know Your Scientist

Henry Cavendish (1731 - 1810) was a British physicist and a natural philosopher. He discovered hydrogen, naming it as 'inflammable air'. In 1798, he conducted an experiment to measure Earth's density using a torsion balance. Cavendish calculated the gravitational attraction between the balls in the apparatus from the period of oscillation of the torsion balance. He then used this value to calculate Earth's density.

Know Your Scientist



Sir Isaac Newton, **(1642-1727)** the English mathematician, astronomer and physicist, was born at Woolsthorpe. He joined Cambridge University in 1661. He became a fellow of Trinity College in 1667 and Lucasian Professor of Mathematics in 1669. He was at the University till1696. His famous treatise *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) was prepared during the years 1665-1666. *The Principia*,as it is commonly known, was not published until 1687.

The theory of gravitation was propounded by one of the greatest physicists, Sir Isaac Newton.

There is a popular story about how Newton discovered the law of gravitation. It is said that when Newton was sitting under an apple tree, an apple fell on his head.

Newton started thinking—'Why did the apple fall down? Why did it not go upwards?'

Newton felt that it could be Earth which attracted the apple toward itself. He theorized that Earth attracted not only the apple, but also the moon, raindrops, stones, etc., toward its surface. He believed that the motion of the apple was accelerated as its velocity changed from zero (when hanging on the tree) to a maximum value (just before it hit the ground). According to his second law of motion, there must be some force acting on the apple that caused this acceleration. He called this force 'the gravitational force' and the associated acceleration as 'the acceleration due to gravity'.

For nearly 300 years, Newton has been considered as the exemplar of modern physical science. His accomplishments in mathematical research are as innovative as those in experimental investigations. He is also known for his works on chemistry, the early history of Western civilisation and theology. Notable among his studies is the investigation of the form and dimensions of the biblical Solomon's Temple.

Did You Know?

Under Earth's gravitational force of attraction, an apple moves towards the ground; but why doesn't Earth move towards the apple? After all, as per Newton's third law of motion, the apple also attracts Earth with the same amount of force.

Newton's second law of motion answers this query. According to this law, the acceleration produced in an object is inversely proportional to its mass. The mass of the apple is negligible compared to Earth's mass. Hence, the acceleration of the apple will be much greater than that of Earth. Therefore, Earth does not move towards the apple.

The gravitational force is responsible for holding the atmosphere above Earth's surface. The moon has no atmosphere because the gravitational force exerted by it is very small.

Importance of the Universal Law of Gravitation

The universal law of gravitation helps us understand several natural phenomena. Some of these are given below.

Dropped objects fall toward the ground.

Earth pulls all objects toward itself through the gravitational force. Hence, when any object is dropped, it falls toward the ground.

The moon revolves around Earth.

The moon is attracted by Earth's gravitational force. This keeps the moon revolving around Earth in its orbit of movement.

The planets revolve around the sun.

The gravitational attraction between the sun and the planets binds the planets in their respective orbits around the sun.

High and low tides occur on Earth's surface.

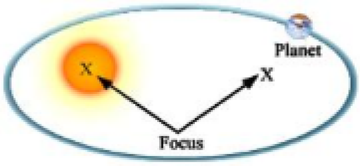
The water present on Earth's surface (in oceans, seas, etc.) is attracted by the gravitational forces of the sun and moon. Hence, the level of water in the seas and oceans rises and falls depending on the relative positions of the sun and moon. This causes high and low tides on Earth.

Did You Know?

Saturn's mass is about 95 times that of Earth, while Jupiter's mass is about 320 times Earth's mass. The mass of the sun is about 333000 times the mass of Earth.

Kepler's Laws of Planetary Motion

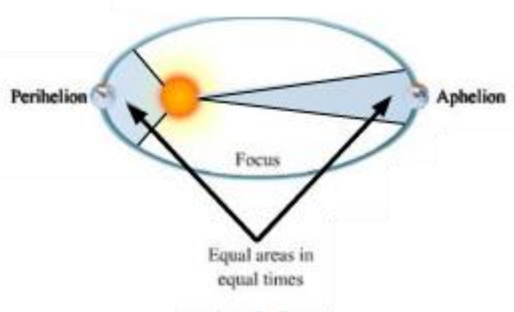
A German astronomer Kepler gave three laws related to planetary motion. These laws are as follows:



First Law: The orbits of the planets are in the shape of ellipse

, having the sun at one focus.

In the figure, the sun is not at the centre of the ellipse. It is at one of the foci marked X. The planet follows the ellipse in its orbit. This means that the distance between a planet and the sun constantly changes as the planet revolves in its orbit.



Second Law: The area swept over per hour by the radius joining the sun and the planet is the same in all parts of the planet's orbit.

In the figure, the imaginary line joining the sun and the planet sweeps out equal areas in equal times. The planet moves faster when it is nearer to the sun. Thus, a planet executes elliptical motion with constantly changing speed as it moves around the sun in its orbit. The point of nearest approach of the planet to the sun is termed perihelion and the point of greatest separation is termed aphelion.

Third Law: The squares of the periodic times of the planets are proportional to the cubes of their mean distances from the sun.

It implies that the time taken by a planet to revolve around the sun increases rapidly with the increase in the radius of its orbit.

Know Your Scientist

In the year 1609, the German astronomer Kepler (having made a careful study of the observations made by Tycho Brahe) came to the conclusion that the orbits of the planets were not circular as had been supposed, but elliptical. He announced his discovery as the following laws.

Johannes Kepler (1571-1630) was a German mathematician, astronomer and astrologer. He discovered the laws of planetary motion. Isaac Newton's theory of Universal Gravitation was later founded upon these laws. Kepler's only self-authored poem that has survived till date is as follows:

Newton and the Inverse-Square Rule

Newton's universal law of gravitation states that the force between two bodies is inversely proportional to the square of the distance between them. Hence, this law is also known as the inverse-square rule. Mathematically, this can be expressed as:

$F \propto 1/r^2$ ...(1)

Newton used Kepler's third law of planetary motion to arrive at the inverse-square rule. He assumed that the orbits of the planets around the sun are circular, and not elliptical, and so derived the inverse-square rule for gravitational force using the formula for **centripetal force**

. This is given as:

$F = mv^2/r$ ...(2) where, m is the mass of the particle, r is the radius of the circular path of the particle and v is the velocity of the particle.

Newton used this formula to determine the force acting on a planet revolving around the sun. Since the mass m of a planet is constant, equation (2) can be written as:

$F \propto v^2/r$ ...(3)

Now, if the planet takes time T to complete one revolution around the sun, then its velocity v is given as:

$v = 2\pi r/T$ ...(4) where, r is the radius of the circular orbit of the planet

or, $v \propto r/T$ ...(5) [as the factor 2π is a constant]

On squaring both sides of this equation, we get:

$v^2 \propto r^2/T^2$...(6)

On multiplying and dividing the right-hand side of this relation by r, we get:

$$v^2 \propto \frac{r^2}{T^2} \times \frac{r}{r} \text{ or } v^2 \propto \frac{r^3}{T^2} \times \frac{1}{r} \ldots(7)$$

Using relation (7) in relation (3), we get

$$F \propto \frac{1}{r^2}$$

Hence, the gravitational force between the sun and a planet is inversely proportional to the square of the distance between them.

Whiz Kid

Earth takes 365 days, 6 hours and 9 minutes to complete one revolution around the sun. The length of the semi-major axis of Earth is 1 A.U. (Astronomical Unit) and the length of the semi-major axis of Mars is 1.52 A.U. **Can you use Kepler's third law of planetary motion to determine the time taken by Mars to complete one revolution around the sun?**

A central force is the force whose magnitude is inversely proportional to the square of the distance from the origin. Thus, gravitational force is a central force.

Solved Examples

Medium

Example 1: **An imaginary planet P, with an orbit of radius *R*, completes one revolution around a star in 64 days. Another planet Q has an orbit of radius 4*R*. How much time will it take to complete one revolution around the same star?**

Solution:

According to Kepler's third law of planetary motion:

$$T_{\mathrm{P}}^2 \propto R_{\mathrm{P}}^3 \quad \ldots . (1)$$

Where, T_{P} is the time period of revolution of P and R_{P} is the radius of the orbit of P

$$T_{\mathrm{Q}}^2 \propto R_{\mathrm{Q}}^3 \quad \ldots . \left(2\right)$$

Where, T_{Q} is the time period of revolution of Q and R_{Q} is the radius of the orbit of Q

On dividing (1) by (2), we get:

$$\frac{T_{\mathrm{P}}^2}{T_{\mathrm{Q}}^2} = \frac{R_{\mathrm{P}}^3}{R_{\mathrm{Q}}^3}$$

$$\Rightarrow T_{\mathrm{Q}} = \left(\frac{R_{\mathrm{Q}}}{R_{\mathrm{P}}}\right)^{\frac{3}{2}} T_{\mathrm{P}}$$

$$\Rightarrow T_{\mathrm{Q}} = \left(\frac{4R}{R}\right)^{\frac{3}{2}} \times 64$$

$$\Rightarrow T_{\mathrm{Q}} = 8 \times 64 = 512 \text{ days}$$

Acceleration Due to Gravity

Acceleration Due to Gravity - An Overview

The force of gravity bounds all objects on or near Earth. When there is force, there must be acceleration. The force of gravity on objects of different masses is different, but the acceleration due to gravity remains the same. What can be the reason for this perplexing behaviour? What are the parameters that determine the acceleration due to gravity of a planet? How does acceleration due to gravity vary with height? When you jump from a height, are you under a free fall?

Let us explore the answers to the what, how and when in the above questions.

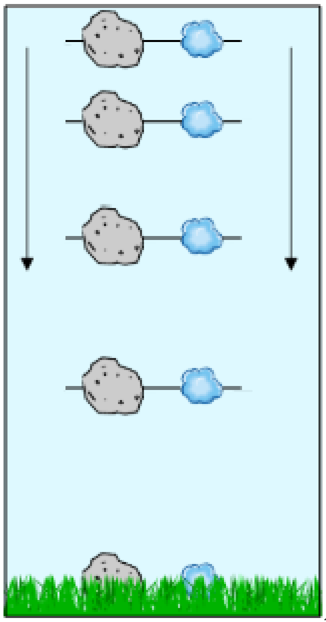
Whenever an object falls towards Earth, it experiences acceleration. This is called **acceleration due to gravity** and is denoted by the letter '*g*'. It is a constant for every object falling on Earth's surface.

- **Acceleration due to gravity does not depend on the mass of the falling object. The value of '*g*' changes slightly from place to place on Earth. The value of acceleration due to Earth's gravity is about 9.8 m/s^2 near Earth's surface.**

Differences between 'G' and '*g*':

Universal gravitation constant (G)	Acceleration due to gravity (*g*)
1. It is defined as the force of attraction acting between two bodies, each of unit mass, whose centres are placed at unit distance from each other.	1. It is defined as the constant acceleration produced in a body when it falls freely under the effect of gravity.
2. Its value is the same throughout the universe.	2. Its value changes from one place to another.
3. It is a scalar quantity.	3. It is a vector quantity.

Free Fall and Acceleration Due to Gravity (*g*)



A free-falling object is an object that falls under the sole influence of gravity. Any object that is being acted upon only by the force of gravity is said to be in a state of free fall.

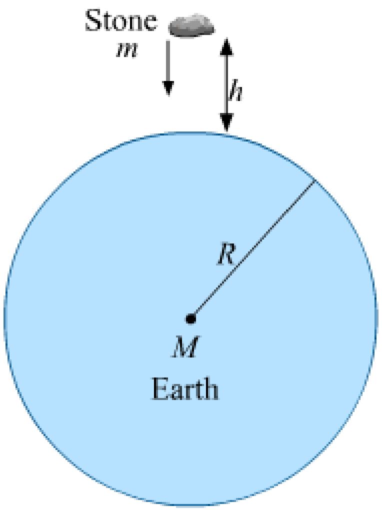
Say, a cotton ball and a large rock are dropped from the same height at the same time. Assuming that air resistance can be eliminated such that neither object experiences any air drag during the course of its fall, **which object will hit the ground first?**

You might say that the rock will hit the ground first, but this is not true. It is generally thought that lighter objects fall slowly, while heavier objects fall rapidly when dropped from the same height. However, objects of different masses fall at the same rate. There is a famous anecdote that Galileo dropped two rocks having different masses from the Leaning Tower of Pisa to show that different objects fall at the same rate, but the credibility of this anecdote is doubtful.

In actual conditions, if you drop a cotton ball and a rock from the same height, then the cotton ball will fall at a slower speed because of air resistance. In ideal conditions, when only gravitational force acts on the cotton ball and the rock (i.e., air resistance is not present), both the objects will fall at the same rate.

This was shown by Robert Boyle when he performed this experiment using a feather and a stone. The feather and stone were put in a tall glass jar and air was removed from the jar using a vacuum pump. When the jar was inverted, both objects fell to the bottom of the jar at the same time. This proved that in the absence of air resistance, all objects fall at the same rate.

Equation for '*g*'



Let us consider a stone of mass *m,* dropped from a height *h*. The stone will fall towards Earth's surface having the mass *M* and radius *R*. This motion of the stone is called a **free fall** under the influence of Earth's gravity.

Free fall is the motion of an object falling solely under the influence of Earth's gravity.

Using Newton's second law of motion, the force on the stone can be given by the product of its mass and acceleration.

$F = ma$

Suppose the stone falls freely with an acceleration *g*.

$F = mg$...(i)

Force exerted by Earth on the stone is given by Newton's law of gravitation:

$$F = G\frac{Mm}{(R+h)^2} \quad \text{...(ii)}$$

From equations (i) and (ii), we obtain:

$$mg = G\frac{Mm}{(R+h)^2}$$

$$\text{Or, } g = \frac{GM}{(R+h)^2} = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

The height h is very small compared to Earth's radius R. Hence, the term $\frac{h}{R}$ will be very small and can be neglected. So, we get:

$$g = G \times \frac{M}{R^2}$$

Where,

G = Universal gravitational constant = 6.67×10^{-11} Nm^2/kg^2

M = Earth's mass

R =Earth's radius

This equation expresses the value of the acceleration due to gravity of an object placed on Earth's surface. This value decreases as we move above from Earth's surface or go below it. Earth's radius R increases when we go from the poles to the equator. Consequently, the value of $\boldsymbol{g}$ decreases.

Solved Examples

Easy

Example 1: **Calculate the value of the acceleration due to gravity on the surface of a planet X having a mass of 5 × 10^{20} kg and the radius as 1800 km. (G = 6.67 × 10^{-11} Nm^2/kg^2)**

Solution:

The formula for calculating acceleration due to gravity is:

$$g = \mathrm{G} \times \frac{M}{R^2}$$

It is given that:

$$\mathrm{G} = 6.7 \times 10^{-11}\ \mathrm{Nm}^2/\mathrm{kg}^2$$

$$M = 5 \times 10^{20}\ \mathrm{kg}$$

$$R = 1800\ \mathrm{km} = 1800 \times 1000\ \mathrm{m} = 1.8 \times 10^6\ \mathrm{m}$$

On putting all these value in the above formula, we get:

$$g = \frac{6.7 \times 10^{-11} \times 5 \times 10^{20}}{(1.8 \times 10^6)^2} = 1.03 \times 10^{-2}\ \mathrm{m/s^2}$$

Example 2: **Calculate the value of acceleration due to gravity on the surface of the moon. The mass of the moon and its radius are 7.4 × 10^{22} kg and 1740 km respectively. The value of universal gravitational constant (G) is 6.7 × 10^{-11} Nm^2/kg^2.**

Solution:

The formula for acceleration due to gravity is:

$$g = \frac{\mathrm{G}M}{R^2}$$

In the present case:

$$\mathrm{G} = \text{Universal gravitational constant} = 6.7 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

$$M = \text{Mass of the moon} = 7.4 \times 10^{22}\ \mathrm{kg}$$

$$R = \text{Radius of the moon} = 1740\ \mathrm{km} = 1.74 \times 10^6\ \mathrm{m}$$

So, the acceleration due to gravity on the surface of the moon is given as:

$$g_{\text{moon}} = \frac{6.7 \times 10^{-11} \times 7.4 \times 10^{22}}{(1.74 \times 10^6)^2} = 1.63\ \mathrm{m/s^2}$$

Example 3:

A block of mass 15 kg falls with an acceleration of 4 m/s^2 on a distant planet. What will be the acceleration of a block of mass 5 kg on the same planet?

Solution:

The acceleration produced by the gravitational force does not depend on the mass of an object. Therefore, the acceleration produced in a block of mass 15 kg will be the same as that produced in a block of mass 5 kg. Hence, both the blocks will fall with the same acceleration, i.e., 4 m/s^2.

Medium

Example 4:

Consider a planet whose mass and radius are each twice as those of Earth. Calculate the acceleration due to gravity on this planet.

Solution:

Let M and M' be the masses of Earth and the planet respectively.

Let R and R' be the radii of Earth and the planet respectively.

The acceleration due to gravity on Earth's surface is:

$$g = \frac{GM}{R^2}$$

On Earth's surface, the value of g is 9.8 m/s^2.

It is given that: $M' = 2M$ and $R' = 2R$

Therefore, the acceleration due to gravity on the surface of the planet is given as:

$$g' = \frac{GM'}{R'^2}$$

$$\Rightarrow g' = \frac{G2M}{(2R)^2}$$

$$\Rightarrow g' = \frac{1}{2} \times \frac{GM}{(R)^2}$$

$$\Rightarrow g' = \frac{9.8}{2}$$

$$\text{So, } g' = 4.9 \text{ m/s}^2$$

Example 5: **The value of acceleration due to gravity at a place is 2% less than its value on Earth's surface. Find the height of that place above Earth's surface. (Given: Earth's radius = 6400 km)**

Solution:

Consider the height of the place above Earth's surface as h.

The formula for the acceleration due to gravity at this place is:

$$g' = \frac{GM}{(R+h)^2} = \frac{GM}{\left(R\left(1+\frac{h}{R}\right)\right)^2}$$

$$g' = \frac{GM}{R^2} \times \left(\frac{R}{R+h}\right)^2$$

$$g' = g\left(\frac{R}{R+h}\right)^2 \qquad \ldots(\text{i})$$

According to the problem, we have:

$$g' = g - \frac{2}{100}g = \frac{98}{100}g \qquad \ldots(\text{ii})$$

On combining equations (i) and (ii), we obtain:

$$\frac{98g}{100} = g\left(\frac{R}{R+h}\right)^2$$

On further solving, we get:

$$h = R\left[\frac{10}{\sqrt{98}} - 1\right]$$

On putting the value of *R*, we get the height of the place as:

h = 65.299 km

Whiz Kid

Why does a parachutist fall down slowly?

Solution:

The surface area of a parachute is large. This increases the air resistance on the parachutist. This air resistance acts in the direction opposite to Earth's gravitational force. Consequently, the parachutist falls down slowly.

SI Unit and Value of '*g*'

Acceleration due to gravity is given as: $g = \frac{GM}{R^2}$ On substituting the SI units of G, M and R in this equation, we obtain the SI unit of acceleration due to gravity.

$$g = \frac{\text{Nm}^2}{\text{kg}^2} \times \frac{\text{kg}}{\text{m}^2} = \frac{\text{N}}{\text{kg}} = \frac{\text{kg.m/s}^2}{\text{kg}} = \text{m/s}^2$$

$$g = \frac{GM}{R^2}$$

We will use the following values of G, M and R in the formula for acceleration due to gravity, in order to obtain the value of g.

$G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

For Earth, $M = 6 \times 10^{24}$ kg

For Earth, $R = 6400\ \text{km} = 6.4 \times 10^6$ m So, we have:

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{\left(6.4 \times 10^6\right)^2}$$

$$g = 9.8\ \text{m/s}^2$$

Earth's acceleration due to the gravity is approximately 9.8 m/s². However, this value varies from place to place.

Know More

Almost all planets are flattened at the poles. This is because of their rotation. This flattening of planets is known as oblateness. The degree of flattening of a planet at its poles is directly proportional to the value of oblateness of the planet. The planets Mercury and Venus rotate extremely slowly. Hence, they are not at all oblate.

Did You Know?

The value of acceleration due to gravity at Earth's centre is zero. This is because the net force of gravity at this place is zero.

Equations of Motion for an Object under the Influence of Earth's Gravity

We have three equations of motion that relate the initial (u) and final (v) velocities of a moving object with its acceleration a along a straight distance s in time t. These equations are given as follows:

First equation of motion: $v = u + at$

Second equation of motion: $s = ut + \frac{1}{2}at^2$

Third equation of motion: $v^2 = u^2 + 2as$

If an object moves only under the influence of gravity, then we can take its acceleration a as the acceleration due to gravity g. Hence, the three equations of motion for acceleration a will be valid for acceleration due to gravity g. These equations are given in the following table.

S. No.	Relation	Object falling downward ($a = g$)	Object moving upward ($a = -g$)
1.	Velocity-time	$v = u + gt$	$v = u - gt$
2.	Distance-time	$s = ut + \frac{1}{2}gt^2$	$s = ut - \frac{1}{2}gt^2$
3.	Velocity-distance	$v^2 = u^2 + 2gs$	$v^2 = u^2 - 2gs$

Solved Examples

Medium

Example 1:

When a ball is thrown vertically upward, it rises to a distance of 20 m. Find the velocity with which the ball is thrown upward. (Take g = 9.8 m/s^2)

Solution:We have:

Initial velocity of the ball = u.

Final velocity (v) of the ball = 0

Acceleration due to gravity,g = −9.8 m/s^2

Height, h = 20 m

On substituting these values in the third equation of motion, we get:

$$v^2 = u^2 + 2gh$$
$$\Rightarrow (0)^2 = u^2 + 2 \times (-9.8) \times 20$$
$$\Rightarrow 0 = u^2 - 392$$
$$\text{So, } u = 19.8 \text{ m/s}$$

Therefore, the ball is thrown upward with a velocity of 19.8 m/s.

Example 2: **A coin is dropped from the top floor of a tall building. The coin takes ten seconds to reach the ground. What is the height of the building? (Take *g* = 9.8 m/s²)**

Solution:

We have:

Initial velocity (u) of the coin = 0

Time taken (t) by the coin to reach the ground = 10 s

Acceleration due to gravity, g = 9.8 m/s^2

Height of the building = h

On substituting these values in the second equation of motion, we get:

$$h = ut + \frac{1}{2}gt^2$$
$$\Rightarrow h = 0 + \frac{1}{2} \times 9.8 \times 100$$
$$\text{So, } h = 490 \text{ m}$$

Example 3: **A cricket ball is thrown upward with some velocity. The ball goes up and comes down in four seconds. What is the velocity with which the ball was thrown? (Take *g* = 9.8 m/s²)**

Solution:

We have:

Initial velocity of the ball = u

Final velocity, v = 0(Since at the highest point, the velocity of the ball is zero)

The total time taken by the ball to go up and come down is four seconds.

So, time taken (t) by the ball to reach the highest point = $\frac{4}{2} = 2\text{s}$

Acceleration due to gravity, $g = -9.8\ \text{m/s}^2$

Using the first equation of motion, we get:

$v = u + gt$

or $0 = u - 9.8 \times 2$

or $u = 19.6$ m/s

Hence, the ball was thrown upward with a velocity of 19.6 m/s.

Example 4: **A gun is fired such that the bullet moves vertically upward with a velocity of 300 m/s. What will be the maximum height attained by the bullet? (Take g = 9.8 m/s^2)**

Solution:

We have:

Initial velocity (u) of the bullet =300 m/s
Final velocity (v) of the bullet = 0 m/s
Acceleration due to gravity, $g = -9.8\ \text{m/s}^2$
Using the third equation of motion, we get:

$$v^2 = u^2 + 2gh$$
$$\Rightarrow 0 = u^2 + 2gh$$
$$\Rightarrow u^2 = -2gh$$
$$\Rightarrow (300)^2 = -2(-9.8) \times h$$
$$\Rightarrow 90000 = 19.6h$$

So, h = 4591.84 m = 4.6 km

Hence, the bullet will attain a maximum height of 4.6 km.

Do you know that the value of 'g' is not constant and depends on various other factors?

Variations in Value of 'g'

1. Change with depth: The value of 'g' varies with the change in the depth. As we move deeper inside the earth, the value of 'g' decreases.
2. Change with height: The value of 'g' is inversely proportional to height. This means that with the increase in the height, there is a decrease in the value of g.
3. Change along the surface of earth: The value of 'g' is not same everywhere on the earth's surface. It is because the earth is not perfectly spherical. The earth is bulged at the equators and flattened at the poles. This means that the radius of earth is greater at the equator and less at the poles. From the following equation we can see that there is an inverse relationship between radius of earth and the value of 'g'.

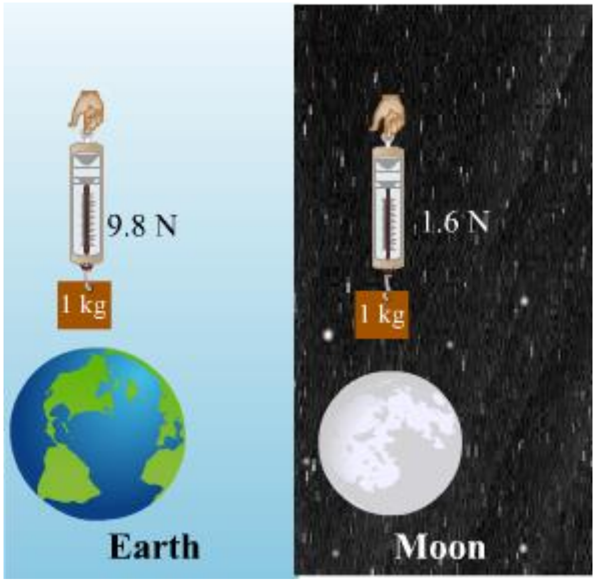
$$g = \frac{GM}{R^2}$$

So, we can infer that the value of 'g' is highest at the poles and lowest at the equator.

Mass and Weight

Mass and Weight: An Overview

Do you know that the mass of a body remains constant? And that the weight of the same body can vary from zero to any finite value, depending upon the celestial body on which it is kept? Have you heard about weightlessness? A body has weight because of gravity; the same body can experience weightlessness under the same gravity. Strange, isn't it?



Suppose a body of mass 1000 kg is placed on Earth's surface and then on the surface of the moon. The mass of the body will remain the same at both places, but it will have different weights (9800 N on Earth and 1600 N on the moon).

The mass of an object is defined as the amount of matter present in it.

It is the measure of the inertia possessed by an object. It is one of the three fundamental physical quantities, the other two being length and time. The mass of an object is usually represented by the small letter 'm'. Its SI unit is kilogram (kg).

The mass of an object is a conserved quantity. It can be neither created nor destroyed during physical or chemical changes. During a physical or chemical process, the total mass of the objects involved remains constant.

Did You Know?

You might know the famous mass–energy equation given by Albert Einstein.

$E = mc^2$

This equation expresses the amount of energy created when mass (m) is lost in a process. The letter 'c' represents the speed of light in vacuum and is numerically equal to 3×10^8 m/s. Let us consider that somehow we are able to completely convert 1 g (= 0.001 kg) of mass into energy. The resultant energy is given as:

$E = 0.001 \times (3 \times 10^8)^2$

$\Rightarrow E = 10^{-3} \times 9 \times 10^{16}$

$\Rightarrow E = 9 \times 10^{13}$ J

This energy is enough to meet the electricity needs of India for more than a year!!

A body contains the same quantity of matter whether it is on Earth, on Mars or in outer space. So, if the mass of an object is 10 kg on Earth, then it will have the same mass on Mars, on the moon and even in outer space. The mass of an object can never be zero.

Weight

The weight of an object is the force of gravity on the object and may be defined as the product of its mass and acceleration due to gravity.

We know that:

Force = Mass × Acceleration

The acceleration produced by Earth's force of attraction is known as acceleration due to gravity and is denoted by the letter 'g'. Thus, the downward force acting on a body is given by:

Force = $m \times g$

Where, m is the mass of the body

By definition, Earth's force of attraction on a body is known as the weight of the body. Hence, on writing 'Weight' (W) in place of 'Force' in the above equation, we get:

Weight, $W = m \times g$

Where, m = Mass of the body

g = Acceleration due to gravity

Weight has the same SI unit as force, i.e., newton (N).

Now, let us calculate the weight of an object having a mass of 1 kg on Earth's surface.

We know that acceleration due to gravity on Earth's surface is 9.8 m/s^2.

Therefore, weight of the object = $m \times g$ = 1 kg × 9.8 m/s^2 = 9.8 N

Know More

Weight has magnitude as well as direction. The weight of a body acts in vertically downward direction and is given by mg. Since the value of 'g' (acceleration due to gravity) changes from place to place, the weight of a body also changes from place to place, i.e., the weight of a body is not constant. In interplanetary space, acceleration due to gravity is negligible. Thus, the weight of a body is zero in interplanetary space, and because of this, one experiences weightlessness.

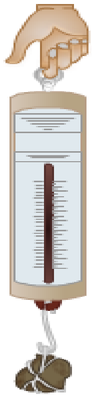
Acceleration due to gravity increases at the poles. As a result, the weight of an object increases at the poles. Acceleration due to gravity decreases at higher altitudes. As a result, the weight of an object decreases at higher altitudes. Acceleration due to gravity decreases under Earth's surface. As a result, the weight of an object decreases under Earth's surface and becomes zero at Earth's centre.

The weight of an object on Earth is the force with which Earth attracts the object toward itself. Similarly, the weight of an object on the moon is the force with which the moon attracts the object toward itself.

Differences Between Mass and Weight

S. No.	Mass	Weight
1.	Mass is the amount of matter contained in a body.	Weight is the force exerted on a body due to the gravitational pull of another body such as Earth, the sun and the moon.
2.	Mass is an intrinsic property of a body.	Weight is an extrinsic property of a body.
3.	Mass is the measure of inertia.	Weight is the measure of force.
4.	The mass of a body remains the same everywhere in the universe.	The weight of a body depends on the local acceleration due to gravity where it is placed.
5.	The mass of a body cannot be zero.	The weight of a body can be zero.
6.	The SI unit of mass is kilogram (kg).	Since weight is a force, its SI unit is newton (N).
7.	The mass of a body can be measured using a beam balance and a pan balance.	The weight of a body can be measured using a spring balance and a weighing machine.

Spring Balance and Beam Balance



While commonly used for measuring the mass of a body, what a spring balance actually measures is the weight of the body (or the force acting in the downward direction).

It can be used locally to measure mass when calibrated correctly according to the value of acceleration due to gravity at the given place.

A spring balance shows different readings on different planets because of the differing values of acceleration due to gravity.

In a spring balance,

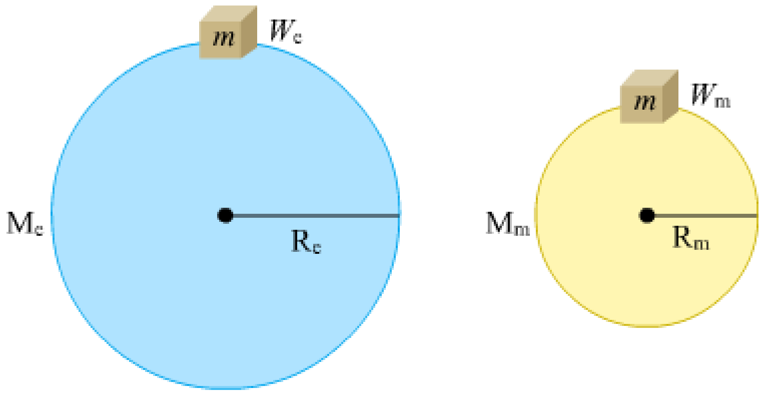
$mg = kx$
Where, x is the extension produced in the spring and k is the spring constant.

$$\therefore x = \frac{mg}{k}$$

So, for differing values of g, x also has different values.

A beam balance is also used for measuring the mass of a body. It does so by comparing the mass of the body with a given standard mass.

Weight of an Object on the Moon



Suppose an object having a mass m and weight W_e on Earth, is brought to the surface of the moon.

So, we have:

Mass of the object = m

Weight of the object on Earth = W_e

Let us take:

Mass of Earth = M_e

Radius of Earth = R_e

Weight of the object on the moon = W_m

Mass of the moon = M_m

Radius of the moon = R_m

Since the mass of the object remains the same everywhere in the universe, it will be the same on both Earth and the moon.

Newton's law of gravitation gives the weight of the object on the moon as:

$$W_{\mathrm{m}} = G\frac{M_{\mathrm{m}} \times m}{R_{\mathrm{m}}^2} \quad \ldots.\left(\mathrm{i}\right)$$

Its weight on Earth is given as:

$$W_{\mathrm{e}} = G\frac{M_{\mathrm{e}} \times m}{R_{\mathrm{e}}^2} \quad \ldots.\left(\mathrm{ii}\right)$$

The values of the mass and radius of Earth and the moon are given in the following table.

	Mass	**Radius**
Earth	5.98 × 1024 kg	6.37 × 106 m
Moon	7.36 × 1022 kg	1.74 × 106 m

Hence, equation (ii) gives the weight of the object on Earth as:

$$W_e = G \times \frac{5.98 \times 10^{24} \times m}{\left(6.37 \times 10^{6}\right)^2}$$

$W_e = 1.4737 \times 10^{11}\ m$ G... (iii)

Equation (i) gives its weight on the moon as:

$$W_m = \frac{G \times 7.36 \times 10^{22} \times m}{\left(1.74 \times 10^{6}\right)^2}$$

$W_m = 2.4309 \times 10^{10}\ m$ G... (iv)

On dividing equation (iv) by equation (iii), we obtain:

$$\frac{W_m}{W_e} = \frac{2.4309 \times 10^{10}\ mG}{1.4737 \times 10^{11}\ mG}$$

$$\frac{W_m}{W_e} = \frac{1}{6}$$

$$\boxed{\frac{\text{Weight of the object on the moon}}{\text{Weight of the object on Earth}} = \frac{1}{6}}$$

From the above result, we can infer that:

- The weight of an object on the moon is one-sixth of its weight on Earth.

- The acceleration due to gravity on the moon is one-sixth of the acceleration due to gravity on Earth.

Solved Examples

Easy

Example 1: **A toy has a mass of 1 kg. Its weight is measured at the equator and at the North Pole using a spring balance. Where do you think the toy would weigh more?**

Solution:

Acceleration due to gravity is more at the North Pole than at the equator. Thus, an object weighs more at the North Pole than at the equator. Hence, the toy will weigh more at the North Pole.

Example 2: **A block of mass 10 kg is taken to the moon. If the acceleration due to gravity on the moon is 1.63 m/s^2, then what is the weight of the block on the moon?**

Solution:

Weight, $W = mg$

Where, m = Mass of the block = 10 kg

g = Acceleration due to gravity on the moon = 1.63 m/s^2

$\therefore W = 10 \times 1.63 = 16.3\ \text{N}$

Example 3: **A horizontal force of 10 N acts on a block weighing 9.8 N. What is the acceleration produced in the block? (Take g = 9.8 m/s^2)**

Solution:

Weight of the block = Mass of the block × Acceleration due to gravity
Let the mass of the block be m.
$\Rightarrow 9.8 = m \times 9.8$
$\therefore m = 1$ kg
Accerleration produced in the block

$$= \frac{\text{Force acting on the block}}{\text{Mass of the block}} = \frac{10\ \text{N}}{1\ \text{kg}} = 10\ \text{m/s}^2$$

Example 4: **Why does the weight of an object change when we move from the poles to the equator?**

Solution:

Earth's radius increases when we move from the poles to the equator. The value of acceleration due to gravity is **inversely proportional** to Earth's radius (R). So, as we move from the poles to the equator, the gravitational force decreases.

$$g = \frac{GM}{R^2}$$

The equation makes it clear that as R increases, the value of g decreases.

Now, the weight of an object is the product of its mass and the gravitational force. So, the weight of the object will decrease as we move from the poles to the equator.

Hard

Example 5: **If a man's weight is 80 N on Earth's surface, then how far must he go from Earth's centre so as to weigh 40 N? (Take Earth's radius = 6400 km)**

Solution:

Weight (W) of the man on Earth's surface = 80 N

The acceleration due to gravity g at height h above Earth's surface is given as:

$g=\frac{GM}{(R+h)^2}$

Weight of the man at height h is:

$$W=mg=\frac{GMm}{(R+h)^2}=\frac{GMm}{\left(R\left(1+\frac{h}{R}\right)\right)^2}$$

$$\Rightarrow W=\frac{GMm}{R^2}\times\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow W=mg\left(\frac{R}{R+h}\right)^2$$

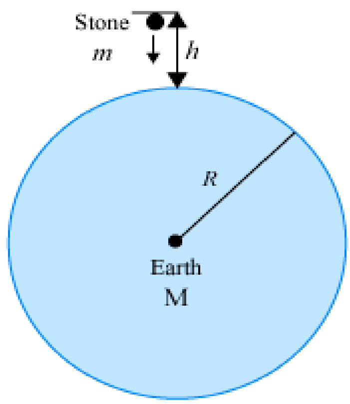
$$\Rightarrow 40=80\times\left(\frac{R}{R+h}\right)^2$$

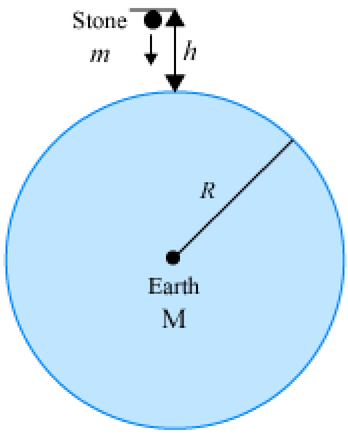
On solving, we get:

$$h=\left(\sqrt{2}-1\right)R=0.414R$$

$$\text{So, } h=2.65\times10^6\text{ m}$$

Therefore, the man must go $(R+h)$ = $(6.4\times10^6+2.65\times10^6=9.05\times10^6)$ m far from Earth's centre so as to weigh 40 N.





Weightlessness

Weightlessness describes the situation wherein the weight of a body becomes zero.

The effective weight of the body at a place (or in a situation) is zero when the effective acceleration due to gravity at that point is zero.

Let us read about the situations wherein the weight of a body becomes zero.

Case I: When the body is taken to Earth's centre

The effective value of acceleration due to gravity at Earth's centre is zero.

Therefore, weight of the body at Earth's centre = $mg' = m \times 0 = 0$

Case II: When the body is revolving around Earth under the influence of the gravitational force

Earth's gravitational pull on the body (acting towards Earth's centre) is balanced by the centrifugal force on the body (acting away from Earth's centre). In consequence, the effective weight of the body becomes zero.

Case III: When the body is inside a lift falling freely under Earth's gravitational force

Acceleration of the lift, $a = g$

Effective acceleration due to gravity = $g' = g - a = g - g = 0$

Hence, effective weight of the body = 0

Solved Examples

Easy

Example: **What is the weight of a body of mass *m* near Earth's surface during its free fall?**

Solution: Weight is a physical quantity that can be experienced only when the body opposes the force of gravity. During free fall, the body does not oppose Earth's gravitational force; hence, its weight is zero.

Did You Know? The motion of a satellite around Earth is an example of free fall. The satellite, at every point, is falling freely toward Earth.

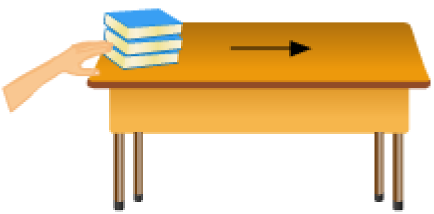
A black hole is formed when a star completely collapses on its gravitational force. A black hole has an intense gravitational field around itself. Nothing can escape from this gravitational field, not even light!

Thrust and Pressure

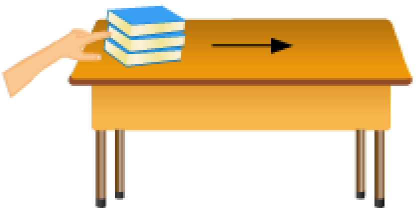
Thrust and Pressure: An Overview

Thrust and **pressure** are two physical quantities related to force. Take, for example, a plastic ball immersed in water. A force is used to submerge it. At the same time, water exerts pressure on the submerged ball. Now, as soon as the force is removed, the upward thrust acting on the ball brings it back to the surface of water.

Thrust and Pressure: In Depth



Case (i)



Case (ii)

Supose we have a pile of three books on a table. If we try pushing the pile with all the fingers of one hand, we will be able to move the books easily. However, this will not be the case if we try pushing the pile with only the index finger.

In 'Case (i)', the effort needed to displace the pile of books is taken care of by the force applied by the fingers. In 'Case (ii)', the force applied by the single finger is not enough; a greater force is needed to displace the books. Thus, the force per unit area exerted by the pile on all the fingers is lesser than that exerted by the same books on the index finger. Consequently, the books move easily in the first case, but not in the second.

This force per unit area is called pressure. It is given by the following relation:

$$\boxed{\text{Pressure} = \frac{\text{Force applied}}{\text{Contact area}} = \frac{\text{Thrust}}{\text{Contact area}}}$$

For a constant magnitude of thrust, if the contact area is greater, then the pressure will be lesser, and vice versa.

Since the SI units of thrust and contact area are N and m^2 respectively, **the SI unit of pressure is pascal (N/m^2)**.

Know More

Heavy vehicles have more than four tyres. Let us understand why this is so.

A wheel of a heavy vehicle has to support a large load. As a result, the consequent pressure on the road due to the wheel is very large. Extra wheels reduce the load carried by the individual wheels, which in turn reduces the pressure on the road due to each wheel. This prevents the wheels from causing damage to the road or sinking into the ground.

Solved Examples

Easy

Example 1: **A force, acting on an area of 0.5 m^2, produces a pressure of 500 Pa. Find the value of the force.**

Solution:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow \text{Force} = \text{Pressure} \times \text{Area}$$

$$= 0.5\ \text{N/m}^2 \times 500\ \text{m}^2 = 250\ \text{N}$$

Example 2: **A force of 100 N is applied on an area of 2 m^2. What is the pressure resulting from this force?**

Solution:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$
$$= \frac{100\ \text{N}}{2\ \text{m}^2} = 50\ \text{N/m}^2$$

Medium

Example 3: **A block of wood has a mass of 20 kg. Its length, breadth and height are 30 cm, 25 cm and 10 cm respectively. On which of its sides should it rest so that it exerts the least pressure on the ground? Also, calculate this pressure. (Take g = 10 m/s^2)**

Solution: The side of the block that has the greatest surface area will exert the least pressure on the ground, and vice versa. Therefore, in order to exert the least pressure on the ground, the block should rest on the side having the dimensions 30 cm × 25 cm.

We can compute the least pressure exerted by the block as follows

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$
$$\text{Force} = \text{Mass of the wooden block} \times g = 20 \times 10 = 200\ \text{N}$$
$$\text{Area} = \frac{30}{100} \times \frac{25}{100} = 0.075\ \text{m}^2$$
$$\text{Pressure} = \frac{200}{0.075} = 2666.7\ \text{N/m}^2$$

Example 4: **Explain why the wheels of an army tank are covered over by a wide steel belt?**

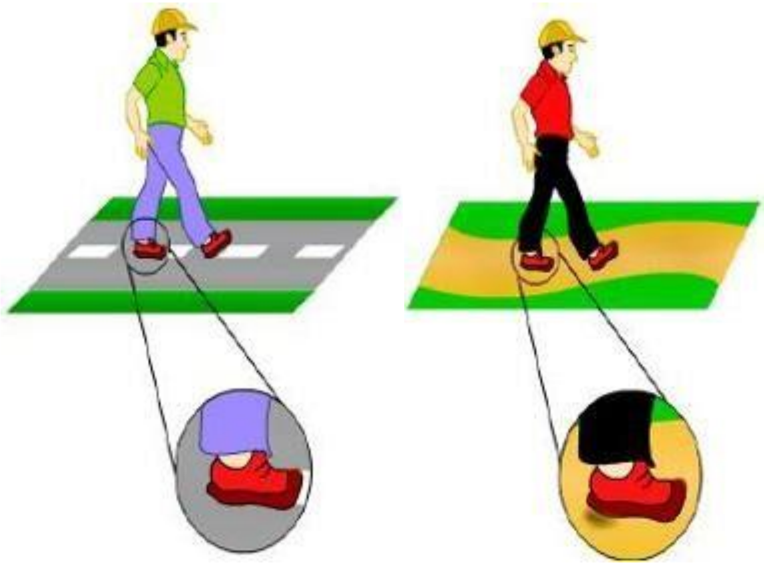
Solution: The steel belt covering the wheels of an army tank has a large surface area. This reduces the pressure exerted by the tank on the ground. As a result, the tank can move easily without damaging the ground or sinking in it.

Know Your Scientist

Blaise Pascal (1623–1662) was a great mathematician and physicist. He worked in the field of geometry and helped in the development of calculators. He also contributed to studies relating to fluids and the pressure distribution in them. The SI unit of pressure is named after him. One pascal is equal to the amount of pressure exerted by a force of one newton on one-square-metre area.

Walking on a Sand Bed

Have you ever wondered why walking on a sand bed is more difficult than walking on a hard road? Let us understand the reason for this phenomenon.



You already know that we push the ground with some force while walking, and the ground in turn applies the same force on our feet. The concrete or soil particles comprising a hard road are tightly bound and immovable. As a result, the reaction force of the ground on our feet is almost equal to the force of our feet on the ground.

A sand bed, on the other hand, consists of loose and movable particles of sand. While walking, these particles get displaced by the force applied by our feet. Consequently, the reaction force of the ground on our feet reduces, which makes walking difficult.

In other words, a hard, rigid surface is able to sustain the pressure applied upon it. Hence, such a surface allows easy movement. However, a soft, loose surface gets deformed under the applied pressure. Hence, such a surface hampers movement.

This phenomenon shows how the same pressure applied by the same force on the same surface area of different surfaces leads to different results.

Applications of Pressure

A few applications of the pressure are discussed here.

1. If you observe a knife used for cutting vegetables, you will notice that the edge of the knife is made very sharp and the area of the edge is very small. Therefore, the pressure on the edge is very high which allows us to cut the materials very easily and with little effort.

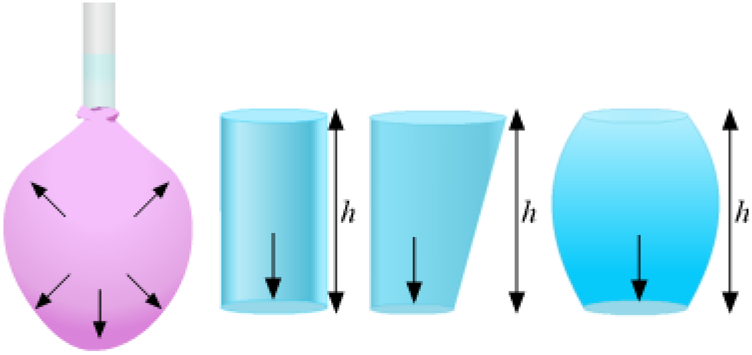
2. The area over which the weight of a skier is distributes is greatly increased by the skis. This reduces the pressure on the snow, and thus, allows the skier to move over snow without sinking into it.

3. While using a straw to drink anything, air is sucked out of the straw. Due to this, the pressure inside the straw is decreased. Hence, the atmospheric pressure outside, forces the liquid to go into the straw.

4. The straps of shoulder bags are generally made broad. The larger area of the strap reduces the pressure on the shoulder of the person who is carrying the bag which makes the bag easier and more comfortable to carry.

Pressure Exerted by Fluids

A fluid is a substance that doesn't have any fixed shape and yields easily to external pressure. Like solids, fluids (liquids and gases) have weight and can exert pressure on the walls of the container in which they are enclosed. When you exert pressure on the surface of a liquid or gas, the pressure is transmitted undiminished through the volume of the fluid in all directions.

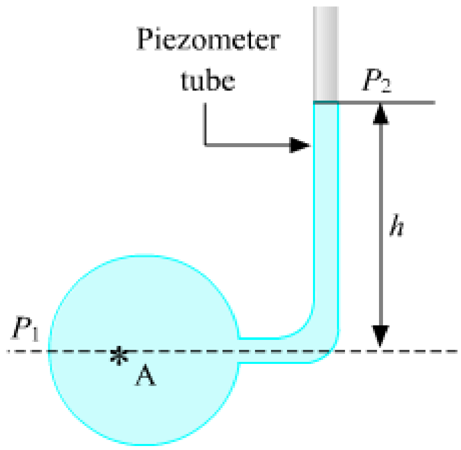
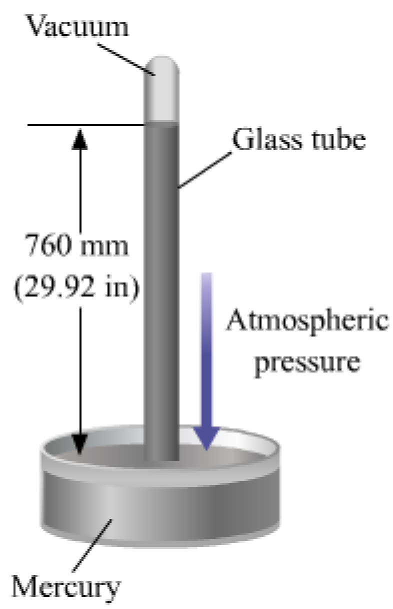


The shape and area of a fluid surface do not affect the pressure exerted by the fluid. It is the height of the fluid column which determines this pressure.

Did You Know?
Air in the atmosphere also exerts pressure. This is known as atmospheric pressure.

Instruments that Measure Pressure

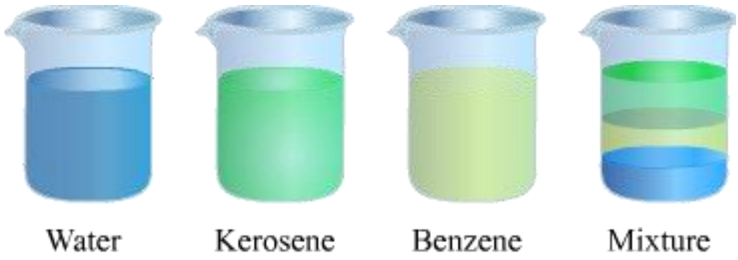
Here are some instruments used for measuring pressure.

		
Bourdon pressure gauge	**Manometer**	**Barometer**

Relative Density

Relative Densities – An Overview

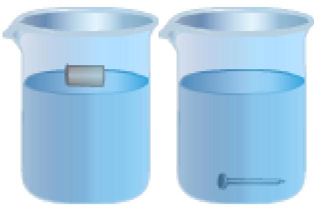
Whenever we use the term ‘relative’ to describe something, a sense of comparison comes to our mind. ‘Relative’ when used with ‘density’ also implies a comparison. It means the comparison of the density of matter.



Knowing the relative density of a substance with respect to that of a liquid helps one figure out whether it will float in the liquid or not. In this lesson, we will learn the concept of relative density in detail.

Question: Have you ever wondered why a cork floats while a nail sinks in water?

Solution: The density of a cork is less than that of water, whereas the density of a nail is greater than that of water. A substance whose density is less than that of a liquid will float on the surface of that liquid. Thus, a cork floats in water. On the other hand, a substance whose density is greater than that of a liquid will sink in that liquid. Thus, a nail sinks in water.



Density

The density of a substance is defined as the mass per unit volume of that substance.

$$\boxed{\text{Density} = \frac{\text{Mass}}{\text{Volume}}}$$

The SI unit of density is kilogram per cubic metre (kg/m^3). Sometimes a smaller unit of density - gram per cubic centimetre (g/cm^3) is also used.

The following table shows the densities of some common substances.

Substances	**Densities (kg/m^3)**	**Substances**	**Densities (kg/m^3)**

Water	1000	Mercury	13600
Kerosene	810	Ice (0°C)	916
Cork	240	Sea water	1025
Iron	7870	Wood	800
Glycerine	1260	Alcohol	790

Know More

The density of the Dead Sea (also called the Salt Sea) is about 1.25 times greater than that of pure water. This density is so high that no human can sink in the Dead Sea. So, one can easily float on the surface of the Dead Sea.

Solved Examples

Easy

Example 1: **The mass of 2 m^3 of aluminium is 5400 kg. Calculate its density in SI unit.**

Solution: It is given that:

Volume of aluminium = 2 m^3

Its mass = 5400 kg

We know that:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\therefore \text{Density of aluminium} = \frac{5400\ \text{kg}}{2\ \text{m}^3} = 2700\ \text{kg/m}^3$$

Relative Density: In Depth

Raju is studying in the light of a kerosene lantern. Suddenly, the light of the lantern goes out because the lower end of the wick is not able to reach the kerosene in the fuel container (as shown in **Figure A**). Not knowing what to do, Raju asks his father. His father tells him to carefully pour some water in the fuel container, making sure that the wick does not come in contact with the water. Raju does as told and is surprised to see the light of the lantern become bright again.

What do you think happens in the fuel container? Does the water start acting as a fuel?

The answer to the second question is NO. Water does not act as a fuel for the lantern. What really happens is this. When poured into the fuel container, the water settles down and causes the kerosene to rise and float on its surface. The lower end of the wick is once again immersed in kerosene and hence, the lantern becomes bright again. A clear partition arises between kerosene and water (as shown **Figure B**). This happens because the density of kerosene (810 kg/m^3) is less than that of water (1000 kg/m^3). In other words, since the relative density of kerosene is less than that of water, it floats in water.

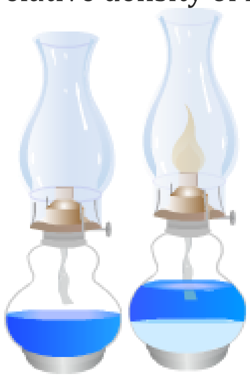


Figure A Figure B

Relative Density: In Depth

The relative density of a substance is defined as its density with respect to that of water (water at 4 ^oC).

$$\text{Relative density of a substance} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

Relative density is also called **specific gravity**. It should be remembered that because **relative density is a ratio of the same physical quantities, it has no unit. It is a pure number.**

The following table shows the relative densities of a few substances.

Substances	Relative densities	Substances	Relative densities

Water	1	Mercury	13.6
Kerosene	0.81	Ice (0°C)	0.916
Cork	0.24	Sea water	1.025
Iron	7.87	Wood	0.8
Glycerine	1.26	Alcohol	0.79
Aluminium	2.7	Gold	19.3

The Relative density of a substance can also be given as the ratio of the mass of the substance to the mass of an equal volume of water at 4 ^{o}C i.e.

Relative density of a substance (R.D.) $= \frac{\text{Mass of substance}}{\text{Mass of an equal volume of water at } 4\,^{o}\text{C}}$

Relative Density of a Solid Substance by Archimedes' Principle

Using Archimedes' principle, we can find the relative density of a solid substance as

$$\text{R.D.} = \frac{W_1}{W_1 - W_2}$$

where W_1 is the weight of the body in air and W_2 is the weight of the body in water.

(1) Relative density of a solid denser than water and insoluble in it

$$\text{R. D.} = \frac{\text{Weight of solid in air}}{\text{Loss in weight of solid in water}} = \frac{W_1}{W_1 - W_2}$$

(2) Relative density of a solid denser than water and soluble in it

$$\text{R. D.} = \frac{\text{Weight of solid in air}}{\text{Loss in weight of solid in liquid}} \times \text{R. D. of liquid}$$

Relative Density of a Liquid Substance by Archimedes' Principle

By definition, relative density of a liquid is

$$\mathrm{R.D.} = \frac{\text{Weight of given volume of the liquid}}{\text{Weight of the same volume of water}}$$

We know by archimedes' principle that if a solid is immersed in a liquid or water, it displaces the liquid or water equal to its own volume.

$$\mathrm{R.D.} = \frac{\text{Weight of a liquid displaced by a body}}{\text{Weight of water displaced by the same body}} = \frac{\text{Weight of the body in air} - \text{Weight of the body in liquid}}{\text{Weight of the body in air} - \text{Weight of the body in water}} = \frac{W_1 - W_2}{W_1 - W_3}$$

Solved Examples

Medium

Example 1: **What is the significance of relative density?**

Solution: Relative density helps us to determine the density of an unknown substance by using the density of a known substance. It enables geologists to calculate the mineral content in rocks.

Example 2: **What are the differences between density and relative density?**

Solution: The density of a substance is defined as the mass per unit volume of that substance. The SI unit of density is kg/m^3.

The relative density of a substance is the ratio of its density to that of a reference material. Usually, the reference material is water. Relative density is also known as specific gravity. It is a pure number, and has no unit.

Archimedes' Principle

Archimedes' Principle: An Overview

'Eureka! Eureka!' Screaming thus, Archimedes came out of his bathtub and ran straight to his king. A popular legend related to the discovery of the principle of buoyancy ends in this manner.

What is this principle of buoyancy? And why is it so important? If you have wondered about this phenomenon, then the following questions must have arisen in your mind.

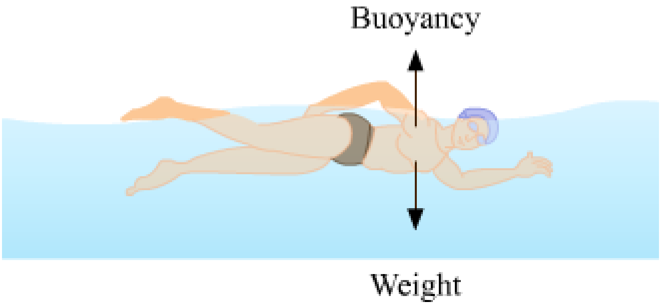
• What has buoyancy to do with the floatation of bodies in liquids?

• Why does a piece of cork rise back to the surface of water even after you force it harder into water?

• Why does a piece of nail made of steel sink but a ship made of the same material float in water?

• Why do you feel lighter while swimming in a pool?

•How can Archimedes' discovery be used in determining the purity of a substance?



Let us go through this lesson to get the answers to all the above questions.

Buoyancy

When an object is immersed partially or fully in a liquid, it experiences an upward force. This **upward force** is known as **buoyant force** and the phenomenon is called **buoyancy**.

When an object is immersed in a liquid, its weight seems to be less than its actual weight. The buoyant force exerted by the liquid is responsible for this phenomenon.

Cause of buoyant force

When a body is partially or fully immersed in a liquid, the displaced **fluid has the tendency to regain its original position due to gravity**. An upward force—called the buoyant force—is, thus, exerted on the body by the displaced fluid.

In equilibrium, the buoyant force is balanced by the weight of the immersed body or the force of gravity acting on it.

The magnitude of the buoyant force acting on the immersed body depends upon two factors.

- Volume of the immersed body
- Density of the liquid

The density of a substance, with respect to the density of a liquid, determines whether the substance will sink or float in the liquid. An iron nail sinks in water because the density of iron is greater than that of water. On the other hand, a cork floats in water as the density of cork is less than that of water. Density is expressed in terms of the volume of a substance. Hence, volume plays a major role in deciding whether a substance will sink or float. Such a relation was given by Archimedes.

Know Your Scientist

Archimedes (287–212 BC) was a Greek mathematician and physicist. According to a legend, he discovered the principle of buoyancy (Archimedes' principle) while taking a bath. It is said that he was so excited with his discovery that he ran naked in the street shouting '*Eureka*'.

Apart from this principle, Archimedes made some very important contributions to the fields of mechanics and geometry. He is considered one of the three greatest mathematicians of all time.

Archimedes' Principle

Archimedes' principle states that **when a body is immersed wholly or partially in a liquid, it experiences an upward buoyant force of magnitude equal to the weight of the liquid displaced by it**.

Buoyant force on an immersed body = Weight of the displaced liquid

Weight of the displaced liquid = Mass of the displaced liquid × Acceleration due to gravity

= Density of the liquid × Volume of the displaced liquid × Acceleration due to gravity

Volume of the displaced liquid = Volume of the immersed body

So,

Weight of the displaced liquid = Volume of the immersed body × Density of the liquid × Acceleration due to gravity

Hence, we can write the magnitude of the upthrust on a body immersed in a liquid as follows:

Buoyant force on an immersed body = Volume of the immersed body × Density of the liquid

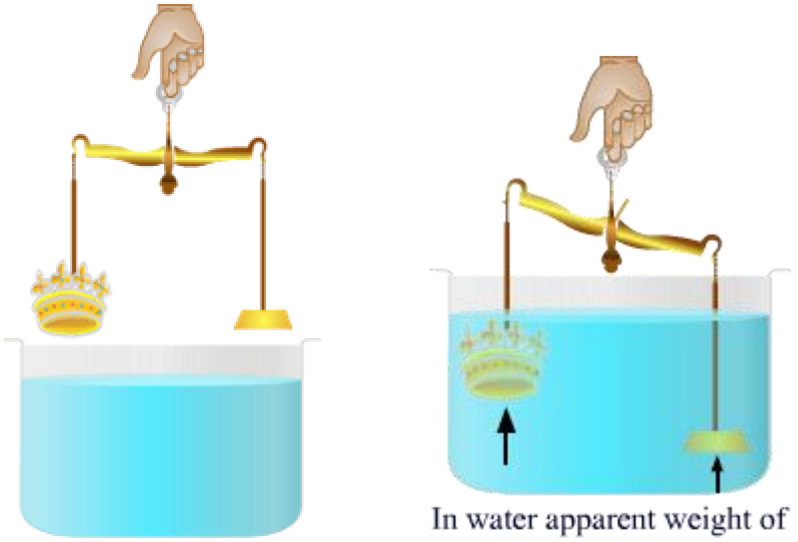
× Acceleration due to gravity

The buoyant force on an immersed body depends on the density of the liquid in which the body is immersed. So, this force is different in different liquids for the same body.

Application of Archimedes' Principle

Archimedes' principle can be used for determining the purity of substances such as gold.

Suppose we have a gold crown and need to determine if it is pure gold or not. We also have a block of pure gold as reference. The block and the crown have the same mass (as shown in the figure). Using Archimedes' principle, we can compare the densities of the crown and the block. If the crown is less dense than the block, then it will displace more water — owing to its greater volume. Consequently, the crown will experience a greater buoyant force than the block (as shown in the figure). This will indicate that the gold used in making the crown is not pure, but has some other metal or alloy mixed in it.

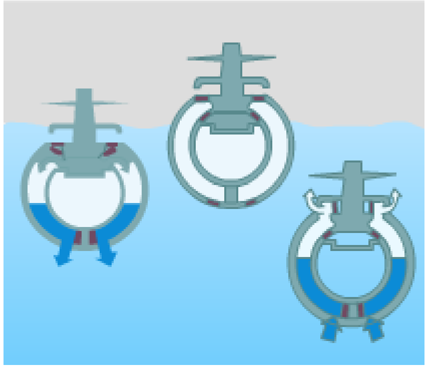


In air weight of the crown is equal to the weight of the piece of pure gold.

In water apparent weight of the crown is less than the apparent weight of pure gold.

Example 1: **Do you know how submarines are made to float or sink as desired?**

Solution: A submarine has large tanks onboard which control how deep it sinks or how high it rises. These tanks are called ballast tanks. To sink the submarine, the tanks are filled with water. The greater the amount of water in the tanks, the deeper does the submarine sink.



To raise the submarine, water is released from the tanks and compressed air (kept onboard in flasks) is let into them. The greater the amount of compressed air in the tanks, the higher does the submarine rise.

Example 2: **When an iron block is dipped in water, it displaces 10 kg of water. Calculate the amount of buoyant force (in Newton) acting on the iron block. (Take g = 9.8 m/s^2)**

Solution: According to Archimedes' principle, the buoyant force on the iron block is equal to the weight of the water displaced by it.

It is given that:

Mass of the water displaced = 10 kg

Acceleration due to gravity = 9.8 m/s^2

∴ Weight of the water displaced = Mass of the water displaced × Acceleration due to gravity

= 10 × 9.8 = 98 N

Hence, the buoyant force acting on the iron block is 98 N.

Medium

Example 3: **How do the densities of an object and a liquid affect the sinking or floating of the object in the liquid?**

Solution: Suppose an object of density ρ and volume V is immersed completely in a liquid of density σ.

Then,

Apparent weight of the object = $W - w = V\rho g - V\sigma g$

Where, W = Weight of the object

w = Weight of the water displaced by the immersed part of the object

Case I: If $W > w$ ($\rho > \sigma$), then $W - w$ is positive.

In this case, the object will sink.

Case II: If $W < w$ ($\rho < \sigma$), then $W - w$ is negative.

In this case, the object will float.

Case III: If $W = w$ ($\rho = \sigma$), then $W - w = 0$.

In this case, the object will rest anywhere within the liquid.

Hard

Example 4:**An object weighs 300 N in air and 150 N in water. Find its relative density.**

Solution:Let us take:

Volume of the object = V

Density of the object = ρ

Density of water = σ_{w}

Acceleration due to gravity = g

It is given that the weight of the object in air is 300 N.

We know that:

Weight of the object = $V\rho g$

So,

$V\rho g$ = 300 N (Neglecting the buoyancy of air)
ρ=300Vg ...(i)ρ=300Vg ...(i)

It is also given that the apparent weight of the object in water is 150 N.

We know that:

Apparent weight of the object = Weight of the object − Weight of the water displaced by it

So,

$$V\rho g - v\sigma_{\mathrm{w}} g = 150$$
$$\Rightarrow 300 - v\sigma_{\mathrm{w}} g = 150$$
$$\Rightarrow \sigma_{\mathrm{w}} = \frac{300-150}{Vg} \quad \ldots.(\mathrm{ii})$$

Now, the relative density of the object can be calculated as follows:

$$\text{Relative density of the object} = \frac{\text{density of the object}}{\text{density of water}} = \frac{\rho}{\sigma \mathrm{w}}$$
$$= \frac{\frac{300}{Vg}}{\frac{300-150}{Vg}}$$
$$= 2$$

Example 5:**An object of density ρ floats in kerosene of density 0.7 × 10^3 kg/m^3 up to a certain mark. If the same object is placed in water of density 1 × 10^3 kg/m^3, will it sink more or less in water?**

Solution:Let us take:

Volume of the object = V

Height of the cross-section of the object = h

Area of the cross-section of the object = A

Height of the object when immersed in kerosene = h'

Height of the object when immersed in water = h''

Acceleration due to gravity = g

It is given that:

Density of the object = ρ

Density of kerosene, $\rho_k = 0.7 \times 10^3$ kg/m^3

Density of water, $\rho_w = 1 \times 10^3$ kg/m^3

According to Archimedes' principle:

Weight of the object = Weight of the kerosene displaced by the object

= Weight of the water displaced by the object

$$\Rightarrow V\rho g = V'\rho_k g = V''\rho_w g$$
$$\Rightarrow (hA)\rho g = (h'A)\rho_k g = (h''A)\rho_w g$$
$$\Rightarrow (h'A)\rho_k g = (h''A)\rho_w g$$
$$\Rightarrow (h'A)\rho_k = (h''A)\rho_w$$
$$\Rightarrow \frac{h'}{h''} = \frac{\rho_w}{\rho_k} = \frac{1\times10^3}{0.7\times10^3} = 1.43$$
$$\text{So, } h' = 1.43h''$$

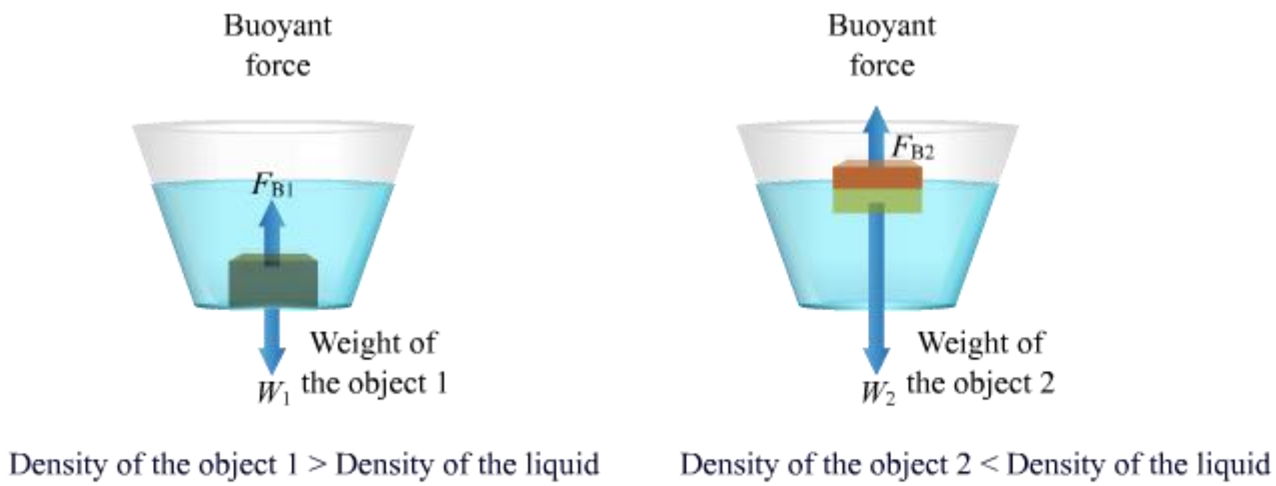
Sinking or Floating

Two forces act on an object placed in a liquid:

- Weight (W) of the object, which acts downwards
- Buoyant force or upthrust (W') exerted by the liquid, which acts upwards

For any object immersed in a liquid:

- If the density of the object is less than the density of the liquid then it will float. The object will be immersed to an extent until the weight of the volume of liquid displaced is equal to the weight of the object
- If the density of the object is more than the density of the liquid then the object will sink as its weight is more than the weight of the liquid displaced.
- If the density of the object is equal to the density of the liquid then it will neither float nor sink in the liquid. It will remain in equilibrium within the liquid wherever it is placed.



Law of Flotation

According to the law of flotation, an object will float in a liquid if its weight is equal to or less than the weight of the liquid displaced by it.

The floating object may be partially or fully submerged in the liquid. Liquid is displaced by the submerged portion of the object.

We can tell whether an object will float or sink in a liquid by comparing its density (or average density) to that of the liquid.

For any object immersed in a liquid:

- If the average density of the object is less than that of the liquid, then the object will float in the liquid.
- If the density of the object is equal to that of the liquid, then the object will float in the liquid but no part of it will be above the surface of the liquid.
- If the density of the object is greater than that of the liquid, then the object will sink in the liquid.